

GCE

Further Mathematics A

Unit **Y532**: Statistics

Advanced Subsidiary GCE

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for AS Level Further Mathematics A

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question		Answer	Marks	AO	Guidance	
1	(i)	Receipt of books is independent <i>OR</i> receipt of one delivery does not affect receipt of another	B1 [1]	1.2	Contextualised (not “events”), properly expressed, no extras	Allow “probabilities independent”
1	(ii)	0.1×0.9^{10} = 0.03486	M1 A1 [2]	1.1 1.1b	Allow 0.1×0.9^{11} Answer, awrt 0.0349	0.03138
1	(iii)	$1 - 0.9^8$ = 0.5695...	M1 A1 [2]	1.1 1.1b	Allow $1 - 0.9^7$ or $1 - 0.9^9$, <i>or</i> add up all terms (± 1 term) Answer, allow 0.570 or 0.57	0.5217 or 0.6126
1	(iv)	$\frac{0.9}{0.1^2}$ = 90	M1 A1 [2]	1.1 1.1b	Correct formula used Exact answer only	
1	(v)	e.g. probabilities different for first few days of year because of holidays	B1 [1]	3.5b	Give contextualised reason for different probability within first few days, or not independent, or not random Needn't state “different probs” or “not independent” explicitly	Not for reason that denies geometric for whole year

Question		Answer	Marks	AO	Guidance	
2	(i)	$0.25 + 0.36 + x + x^2 = 1$ $x^2 + x - 0.39 = 0$ oe $x = 0.3$ (or -1.3) x cannot be negative $E(W) = 2.23$ $E(W^2) = \sum w^2 p(w)$ [= 5.83] Subtract $[E(W)]^2$ to get 0.8571 AG	M1 A1 A1 B1ft B1 M1 A1 [7]	3.1a 1.1b 1.1b 2.3 1.1b 1.1 2.1	Equation using $\sum p = 1$ Correct simplified quadratic Correctly obtain $x = 0.3$ Explicitly reject other solution 2.23 or exact equivalent only Use $\sum w^2 p(w)$ Correctly obtain given answer, www	Can be implied Method needed ft on their quadratic Allow for $E(W)^2 = 4.9729$ Need 2.23 or 4.9729 and 5.83 or full numerical $\sum w^2 p(w)$
2	(ii)	$9 \times 0.8571 = \mathbf{7.7139}$	B1 [1]	1.1b	Allow 7.71 or 7.714	
3	(i)	Flaws must occur at constant average rate (uniform rate)	B1 [1]	1.2	Context (e.g. “flaws”) needed Extra answers, e.g. “singly”: B0	Not “constant rate” or “average constant rate”.
	(ii)	$Po(2.1)$ or $e^{-\lambda} \frac{\lambda^3}{3!}$ $= \mathbf{0.189}$	M1 A1 [2]	1.1 1.1b	$Po(2.1)$ stated or implied, or formula with $\lambda = 2.1$ stated Awrt 0.189	
	(iii)	$Po(3)$ $1 - P(\leq 3)$ $= \mathbf{0.3528}$	M1 M1 A1 [3]	1.1 1.1 1.1b	$Po(2 \times 0.7 + 1.6)$ stated or implied Allow $1 - P(\leq 4) = 0.1847$, or from wrong λ Awrt 0.353	Or <u>all</u> combinations ≤ 3 1 – above, not just = 3

Question		Answer	Marks	AO	Guidance
4	(i)	0.4(00) BC	B2 [2]	1.1 1.1b	SC: if B0, give SC B1 for two of $S_{xx} = 12500$, $S_{yy} = 1600$, $S_{xy} = 1790$ and $S_{xy}/\sqrt{(S_{xx} S_{yy})}$ Also allow SC B1 for equivalent methods using Covariance & SDs
4	(ii)	Data needs to have a bivariate normal distribution	B1 [1]	1.2	Needs “bivariate normal” or clear equivalent. <i>Not</i> just “both normally distributed” Allow “scatter diagram forms ellipse”
4	(iii)	H_0 : higher maths scores are not associated with higher BCF grading; H_1 : positively associated CV 0.3783 $0.400 > 0.3783$ so reject H_0 Significant evidence that higher maths scores are associated with higher BCF grading	B1 B1 M1ft A1ft [4]	2.5 1.1b 2.2b 3.5a	Needs context and clearly one-tailed <i>OR</i> ρ used and defined <i>Not</i> “evidence that ...” Allow 0.378 Reject/do not reject H_0 Contextualised, not too definite Needn’t say “positive” if H_1 OK SC 2-tail: B0; 0.4438, or 0.3783 B1; then M1A0 $H_0: \rho = 0$, $H_1: \rho > 0$ where ρ is population pmcc (<i>not</i> r) FT on their r , but <i>not</i> CV <i>Not</i> “scores <u>are</u> associated ...”. FT on their r <u>only</u>
4	(iv)	It makes no difference as this is a linear transformation	B1 [1]	2.2a	Need <i>both</i> “unchanged” <i>oe and</i> reason, need “linear” or exact equivalent “oe” includes “their 0.4”

Question		Answer	Marks	AO	Guidance													
5	(i)	$({}^8C_6 \times {}^{12}C_3) + ({}^8C_7 \times {}^{12}C_2) + (1 \times {}^{12}C_1)$	M1	1.1	Any one correct pair of nC_r multiplied	Multiplication method: all possibilities M2 All correct A1, Answer A1												
		$[= 28 \times 220 + 8 \times 66 + 12 = 6160 + 528 + 12]$	M1	1.1	Three pairs added													
		$\div {}^{20}C_9 \quad [= 167960]$	B1	1.1	${}^{20}C_9$ or 167960 seen anywhere													
		$= \frac{335}{8398}$ or 0.039890...	A1 [4]	1.1b	Exact or awrt 0.0399													
	(ii)	$2 \times (n-1)!$	M1	3.1a	SC: any other method: $\frac{2}{n}$ B2 Compare their $\frac{2}{n}$ with 0.1, needs division <i>Not</i> ≥ 21 SC: $\frac{2}{n+1}$, 22: B1 + B1 SC: $\frac{1}{n}$, 11: B1 + B1	T&I: any one of the below: B2 <table border="1" data-bbox="1720 624 2063 711"> <tr> <td>19</td> <td>20</td> <td>21</td> <td>22</td> </tr> <tr> <td>0.10</td> <td>0.1</td> <td>0.09</td> <td>0.09</td> </tr> <tr> <td>5</td> <td></td> <td>5</td> <td>1</td> </tr> </table> Final answer 21: B2 Ignore inequalities until final answer only	19	20	21	22	0.10	0.1	0.09	0.09	5		5	1
		19	20	21			22											
		0.10	0.1	0.09			0.09											
		5		5			1											
		$\div n!$	M1	1.1														
$\frac{2}{n} < 0.1$	M1	1.1																
$n_{\min} = 21$	A1 [4]	3.2a																
6		DR				Allow q for $1-p$												
		$np = 5.625$	B1	1.1														
		$np(1-p) = 1.875^2 \quad [= 3.515625]$	B1	1.1	Allow $np(1-p) = 1.875$													
		$\Rightarrow (1-p) = \frac{5}{8}$ or 0.625	M1	3.1a	Eliminate one letter													
		$\Rightarrow p = \frac{3}{8}$ or 0.375	A1	3.2a	Or exact equivalent													
$n = 15$	A1 [5]	1.1b	Exact answer only															

Question		Answer				Marks	AO	Guidance						
7	(i)	Neither				B1 [1]	2.5	OE	Not “neither is independent of the other”					
	(ii)	$c = 2.848 - 0.1567m$ BC				B1 B1 B1 [3]	1.1 1.1 1.1	Correct a , awrt 2.85 Correct b , awrt 0.157 Letters correct from correct method (If both wrongly rounded, e.g. $c = 2.84 - 0.156m$, give B2)	SC: m on c : $m = 15.65 - 4.832c$: B2 $y = 15.65 - 4.832x$: B1 $c = 15.65 - 4.832m$: B1 If B0B0, give B1 for correct letters from valid working					
	(iii)	a unchanged, b multiplied by 2.2 (allow “ a unchanged, b increases”, etc)				B1 [1]	2.2a	oe, e.g. $c = 2.848 - 0.345m$; $m = 7.114 - 2.196c$	SC: m on c in (ii): Both divided by 2.2 B1					
	(iv)	Draw approximate line of best fit Draw at least one vertical from line to point Say that “Best fit” line minimises the sum of squares of these distances				M1 M1 A1 [3]	1.1 2.4 2.4	Needs M2 and “minimises” and “sums of squares” oe	SC: Horizontal(s): full marks (indept of (ii))					
8		<table border="1"> <tr> <td>Expected frequency</td> <td>16</td> <td>8</td> <td>16</td> </tr> <tr> <td>$\sum \frac{(O-E)^2}{E}$</td> <td>$\frac{4^2}{16}$</td> <td>$\frac{(y-8)^2}{8}$</td> <td>$\frac{(12-y)^2}{16}$</td> </tr> </table> <p>$X^2 > 5.991 \Rightarrow$ $1 + \frac{(y-8)^2}{8} + \frac{(12-y)^2}{16} > 5.991$ $\Rightarrow 3y^2 - 56y + 192.144 > 0$ Critical values 4.53 and 14.14 $(0 \leq) y \leq 4$ or $15 \leq y \leq 28$</p>	Expected frequency	16	8	16	$\sum \frac{(O-E)^2}{E}$	$\frac{4^2}{16}$	$\frac{(y-8)^2}{8}$	$\frac{(12-y)^2}{16}$	M1 A1 M1* depM1 B1 M1 M1 A2 [9]	3.1a 3.3 2.1 3.4 1.1b 1.1 1.1 2.2a 3.5b	Method for expected frequencies Expected frequencies all correct Use $\sum \frac{(O-E)^2}{E}$ $X^2 > CV$ Correct CV, ft on cells combined Simplified 3-term quadratic Solve quadratic to obtain CVs www. One number wrong or omitted: A1	Use of cc (Yates?) loses A marks only SC: 8+4+8: M0A0 M1M1B1 M1M1A0A0 Allow = but not $X^2 < CV$ 8+4+8: 3.841 (not 5.991) Can be implied Ignore $0 \leq y$
Expected frequency	16	8	16											
$\sum \frac{(O-E)^2}{E}$	$\frac{4^2}{16}$	$\frac{(y-8)^2}{8}$	$\frac{(12-y)^2}{16}$											

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